Subitizing and Visual Counting in Children with Problems in Acquiring Basic Arithmetic Skills

Burkhart Fischer, Dipl. Phys., Christine Gebhardt, Dipl. Phys., and Klaus Hartnegg, Dipl. Phys.

Center of Neuroscience, Optomotor Laboratory, University of Freiburg, Freiburg, Federal Republic of Germany

ABSTRACT

The ability of recognizing a number of briefly presented items without actually counting is called subitizing (from lat. subito = suddenly). Adult subjects can subitize 3 to 4 items. For greater numbers the subjects begin a counting process relying on the visual memory of the test pattern, which needs increasingly more time as the number of items increases. The development of accuracy and speed of subitizing and visual counting was measured for subjects up to the age of 17 years. Furthermore, this study tests the hypothesis that children with difficulties in acquiring basic arithmetic skills exhibit developmental deficits in subitizing and/or counting. The study does not intend to investigate theories on the nature of dyscalculia even though most test children can be classified as dyscalculic.

Methods: Two-hundred-nineteen control subjects and 156 test subjects with problems in arithmetic skills in the age range of 7 to 17 years were given a visual counting task in which 1 to 9 items were presented for 100 ms. The subjects had to press a digit key on a numerical keyboard to indicate the number of items they had seen. Percentages of correct responses and response times were recorded.

Results: The analysis shows systematic differences between control and test children increasing with age. The percentage of test children performing below the 16-percentile of the age matched controls was estimated to be between 40% and 78% (increasing with age).

Conclusions: We concluded that the deficit in a basic visual capacity may contribute to the problems encountered by children with anomalies in acquiring basic arithmetic skills.

Keywords: subitizing, visual number counting, dyscalculia, saccades

Introduction

The human brain has the capacity of correctly recognizing the number of items that have been presented for no less than 100m. This presentation time is too short to count the items by using scanning saccades. The item number is either recognized immediately without a conscious process of counting (subitizing for item numbers up to 4) or by counting the items using visual memory[1, 2]. The subitizing part of the process is characterized by a response time, which is about the same (or slightly increasing) for 1 to 3 or 4 items. In the counting part of the process, linearly increasing response times are observed indicating that for each additional item extra time is needed to find the correct number[3, 4].

The question whether subitizing and counting are two separate processes in the brain is still being discussed[4, 5, 3, 6, 2]. A positron emission tomography (PET) study has shown, that both processes use the same structures of the extra striate middle occipital and intraparietal area[4]. Using fMRI it has been shown, that within the parietal cortex there exist three different circuits for number processing[7].

It has been shown, that saccadic control, as well as auditory discrimination undergoes a long lasting development during which the variables describing the quality of the performance of specific tasks keep changing until the age of 18 years[8, 9]. This implies that

---

Correspondence regarding this article can be emailed to burkhart.fischer@blicklabor.de or sent to Prof. B. Fischer, Optomotor Laboratory, Hans-Sachs-Gasse 6, D-79098 Freiburg, Federal Republic of Germany.

Fischer B, Gebhardt C, Hartnegg K. Subitizing and Visual Counting in Children with Problems in Acquiring Basic Arithmetic Skills. Optom Vis Dev 2008:39(1):24-29.

children at the beginning of school are far from the performance level of adults. The question, whether subitizing and counting are also subjected to such a long lasting development is treated in the first part of this paper.

It has been speculated that the basic visual capacity of subitizing is used by the brain to develop a sense of number, which allows associating the number words (the auditory signal) and the digits (the visual signal) with the number of items for which they are used[10]. It was shown, that dyscalculia is not a consequence of low intellectual cognitive performance but rather relatively basic numerical capacities possibly including subitizing were made responsible for the selective deficit in children with dyscalculia[11,12]. Comparing control children with children with dyscalculia fRMI studies show differences in the activation of frontal and parietal structure during approximate calculation[13].

This study addresses this question by determining the performance on a standardized visual counting task given to control children and to children, who have specific problems in acquiring basic arithmetic skills, who for example, use their fingers to add 3 to 4 still at an age of 8 years or older. This problem often occurs specifically in dealing with numbers but not in acquiring reading or spelling skills. In this case the problem is known as dyscalculia given that the general intelligence is also normal (IQ >80).

The data show the developmental deficits by using this diagnostic task: the development is considerably slower in the test children as compared to the controls. In the companion paper (in this issue of *Optometry & Vision Development*), it is shown that the visual capacity of subitizing and counting can be improved by daily practice and that this improvement results in a significant gain in acquiring basic arithmetic skills.

Methods

Subjects: The 375 subjects of the total group were 7 to 17 years old. They were divided into 4 different age groups as shown in Table 1. Controls (N=219) were recruited from schools in Freiburg and the vicinity. They were selected on the basis of their general performance at school and showed no indications for dyslexia, attentional deficits hyperactivity disorder, or dyscalculia as indicated by their grades. The children of the test group (N=156) demonstrate poor performance in exhibited arithmetic skills (as determined by the Zareki test or DEMAT available from the Testzentrale in Göttingen, Germany) but reached normal reading and spelling levels. General intelligence (IQ) was measured by the Kaufmann ABC or the HAWIK test. Children with IQs below 80 were excluded from the study.

Visual stimuli and task: One to 9 small circles (items) were presented simultaneously on a small LCD display (2,5cm x 6cm, outer borders) corresponding to 5.7 x 11.4 deg of visual angle at a viewing distance of 30cm. The circles were 2mm in diameter (0.04 deg) and were presented in black against a greenish background. The stimuli and their contrast were too small to give rise to after images. The minimum distance between the stimuli was 8mm horizontally and 5mm vertically. The spatial positions of the randomly selected number items were randomized within a 4 x 4 array. (Fig.1 shows examples of the presentation of 4, 6, and 8 items.) By chance, some of the presentations looked regular (like the numbers on a die) others looked irregular.

A central fixation mark was presented in the centre of the display at the beginning of each trial. It was turned off when the items were presented. Therefore, all stimuli were presented parafoveally. The items were presented for 100ms (the shortest possible

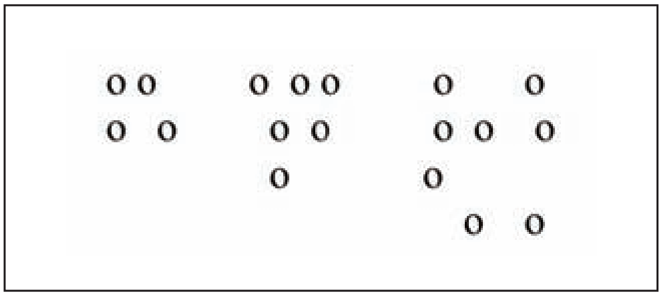

Figure 1: *Typical examples of presentations of 4, 6, and 8 items.*

	7-8 Years	9-10 Years	11-13 Years	14-17 Years	All
Controls					
Number female / male	35 (18f/17m)	36 (20f/16m)	62 (36f/26m)	86 (54f/32m)	219 (128f/91m)
Mean age (y)	7,5 +/- 0,5	9,6 +/- 0,5	11,9 +/- 0,8	15,4 +/- 1,0	
Test Subjects					
Number female/male	20 (14f/6m)	60 (32f/28m)	67 (32f/35m)	9 (1f/8m)	156 (79f/77m)
Mean age (y)	7,6 +/- 0,5	9,6 +/- 0,5	11,7 +/- 0,8	15,6 +/- 1,2	

Table 1: *The table shows the number of female and male subjects in both groups being divided into 4 age groups. Mean age and standard deviations are given. Altogether 375 (207f/168m) participated in this study.*

saccadic reaction time of human subjects[14]) to prevent any saccade that the subjects could possibly make to count the stimuli by scanning saccades. A longer but limited exposure time would confound enumeration by saccades with subitizing[15]. The limited presentation time did not imply a limited time to find the response (see below).

The visual display, the keyboard, and data collection were all implemented in a small hand held instrument. The data stored in the instrument were down loaded and formed the data base for the analysis presented in this paper.

Procedure: The subjects were introduced to the task by presenting each set of items for an unlimited period of time. They were instructed to press the key corresponding to the number of items at the display. The spatial arrangement of the digit keys was identical to those of the numerical keyboards of computers. At least 12 practice trials of this kind were given. Subjects were informed that reaction time was important. Therefore they were also instructed to place their hand above the center of the keyboard. After they understood the task all subjects were given another 5 trials of the real task for practice with a presentation time of 100 ms.

The central fixation point was presented first. After 1 sec the fixation point disappeared, the test pattern was presented, and the subject responded by pressing the response key. The next trial was initiated only after another key press by the subject. Even though reaction time was important the speed of the complete task performance could be controlled by the subject. Each number of items was shown 20 times with the exception of a single item, which occurred only 10 times. Altogether, 170 trials were run for each subject. The total time for a test session was about 20 min.

Data collection and analysis: The data were recorded by the test instrument and downloaded to a personal computer for later analysis. For each subject and each trial the number of items (N) presented and the digit number (n) of the pressed response key were recorded together with the response time r between the end of the presentation to the key press. In the analysis presented below only correct trials were used to calculate the mean reaction times.

For each subject and for each number of items we calculated the percentage of correct responses p(N) and the corresponding mean response time r(N). For N > 3 an almost linear relationship was obtained for r(N) as a function of N. Therefore we calculated the linear regression between the response times and the item number for N=4 to N =8. The slope of the regression line is the mean of the extra time t needed for each additional item. The basic reaction time T was calculated as the mean value of r(1). The mean percentage of the correct responses P was calculated for item numbers between 4 and 8. Item number 9 was excluded from this analysis, since most children noticed that 9 was the highest number on the key

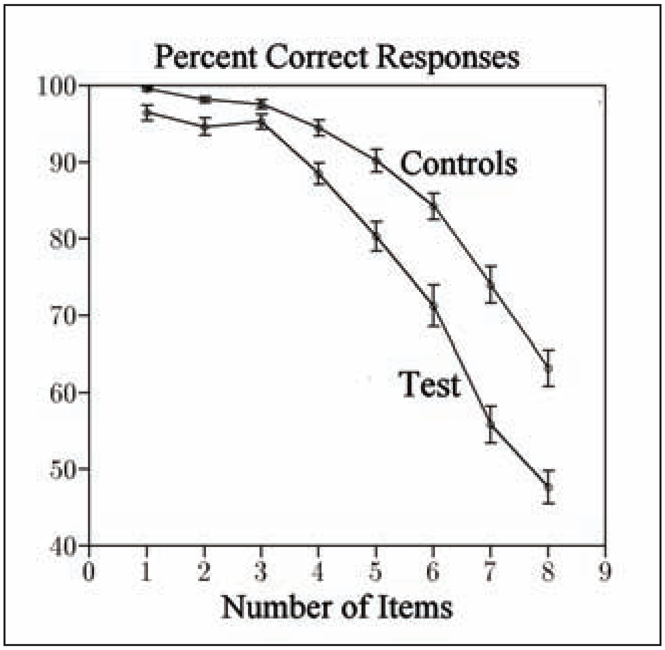

Figure 2: *Percentages of correct responses as a function of item number. In this and the following figures the vertical bars indicate standard errors.*

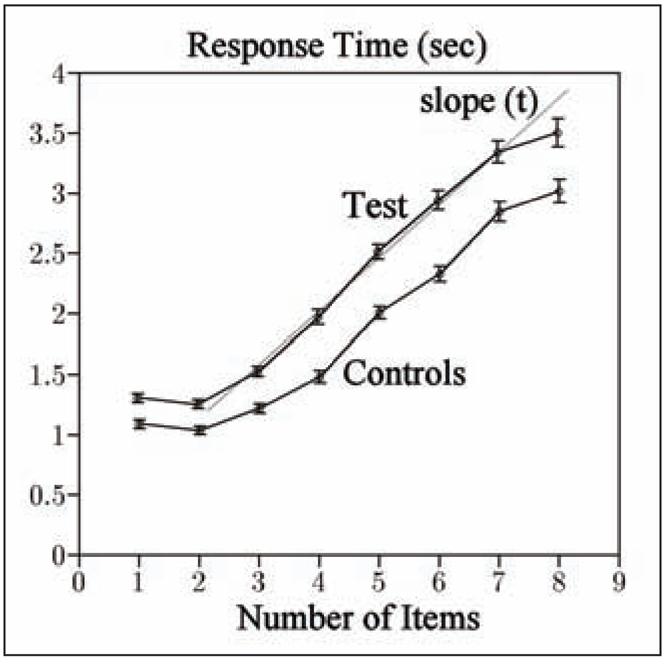

Figure 3: *Mean response times as a function of item number. The linear regression (dotted) line has the slope t (defining the time per item).*

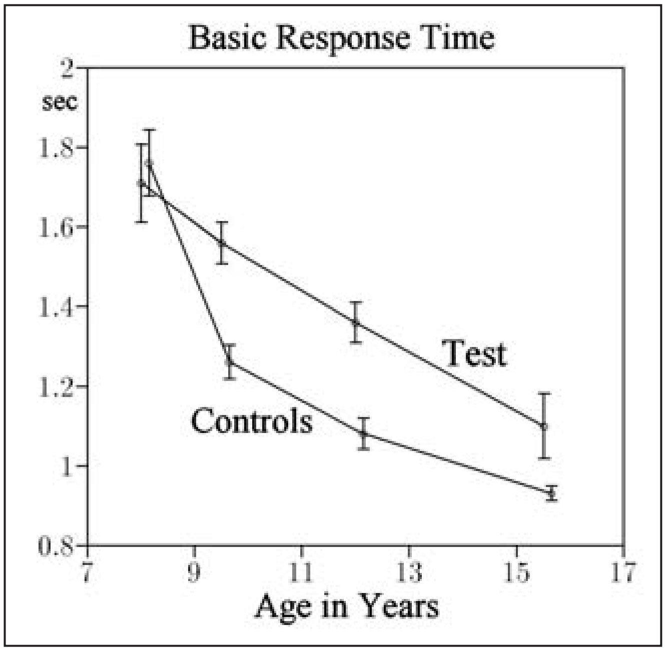

Figure 4: *The decrease of the basic response time T is shown as a function of age for both groups of subjects.*

board and pressed 9, whenever the item number was "large". They hit the correct key "9" often by guessing. We also calculated these values separately for regular and irregular patterns and found slight differences only for item numbers above 5. These slight profits were similar in both groups and therefore neglected in the final analysis.

To compare the performance of the control and the test subjects, age curves of the characteristic variables were calculated. The characteristic variables were: the basic response time T; the time per item t; the correctness Pas defined by the percentage of correct responses for item numbers 4 to 8. We also calculated the effective recognition speed by dividing P by t. ANOVAs were used to determine the significance of the differences between groups and the effects of age.

Results

Count curves: The typical form of count curves of a group of controls aged 11-13 years can be seen in Fig.2. High percentages of correct responses were obtained for item numbers below 4. For larger item numbers a clear decline can be seen. For 8 items about 65% of the responses were still correct. The test children showed a qualitatively similar response curve, but the curves show smaller values of correct responses. Note that even for small item numbers of 1 to 3 about 5% errors were made by the test subjects. With increasing item number the differences between the curves increased.

Response times as a function of item number are shown in Fig.3 for the same age groups as in Fig.2. Again the curves have similar forms, but the curves for test children were displaced vertically to longer response times. Note that even for small item numbers 1 and 2 the test children were slower than the control children. With increasing item number the response times became longer in an almost linear fashion indicating that both groups needed a constant additional time for each additional item given by the slopes t of the linear parts of the curves.

Age curves: Count Curves (as shown in Fig.2 and 3 for age group 3) were obtained for all age groups. To examine at the age effects the characteristic variables were calculated for each curve (see Methods). Fig.4 shows the age curves of the basic response T for both groups. The developmental decrease can be seen in both groups, but the test subjects showed a systematically slower development. While the differences were close to nothing for the youngest age group they were highly significant for the age groups 2 and 3. Age group 4 contained only 9 test subjects with a large inter-individual scatter. The difference between control and test subjects in this age group did not reach significance. The ANOVA resulted in F=60.15 (p=0.0001) and with age as covariate we obtained F=25.8 (p=0.0001). The linear correlation coefficient was r = - 0.616.

Fig.5 shows the age curves for correctness P and the time per item t describing the counting part of the process. The discrepancy in both pairs of curves shows that the test subjects reached lower correctness with lower speed than the controls. They could not use the longer response times to reach as many correct responses as the control children. On the contrary, they took more time for each additional item and made more mistakes than the controls. The ANOVA for the variable P resulted in F=77.51 (p=0.0001) and with age as covariate F=48.22 (p=0.0001). The correlation coefficient was r=0.342. ANOVA results for the variable t were F=79.1 (p= 0.0001) and with age as covariate F=38.03 (p=0.0001), and r = -0.513.

Percentage of off-limit subjects: In practice it is of interest to know how many subjects from a test group failed to reach the criterion of a control group. For an estimate we used the effective recognition defined by P/t, which combines the correctness and the time per item calculated for item numbers 4 to 8. Fig.6 shows the two age curves. The numbers indicate

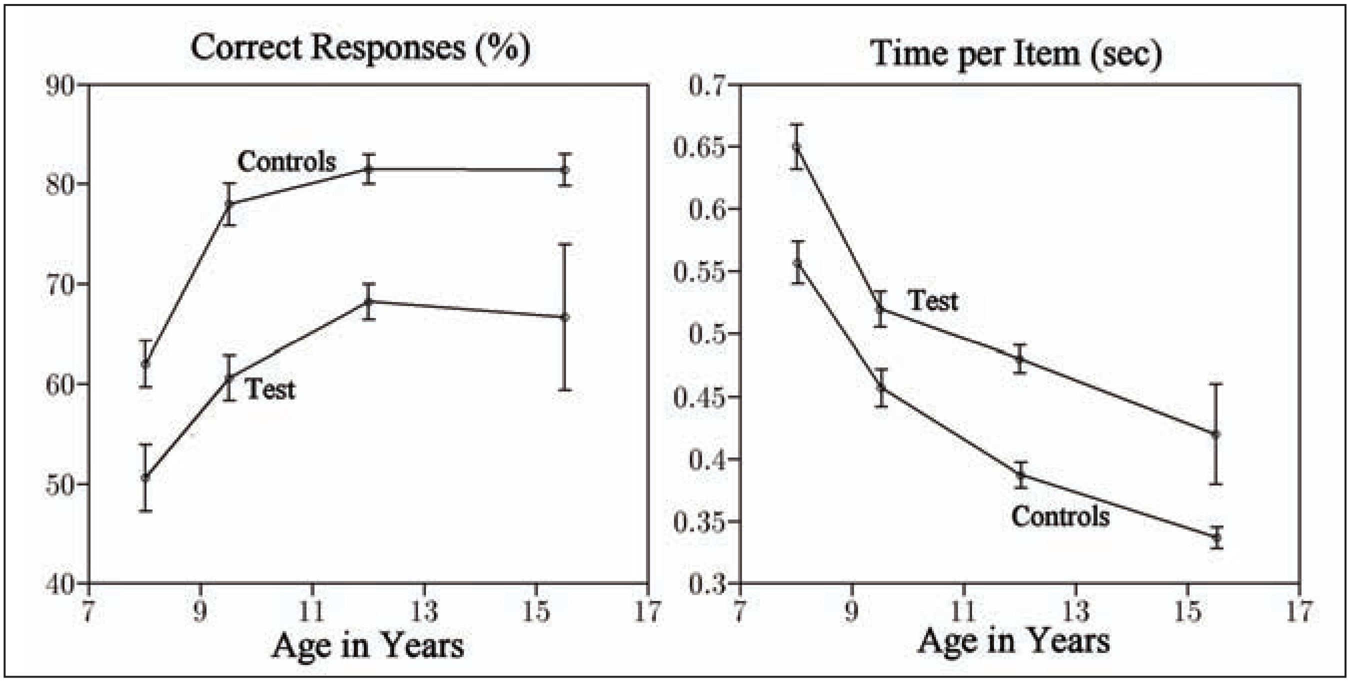

Figure 5: *The percentage of correct responses (left) and the time per item (right) are shown as a function of age for both groups of subjects.*

the percentages of test subjects performing below the 16-percentile of the controls.

The figure shows that these percentages among the test subjects increase with increasing age. While in the youngest group about 40% of the test subjects failed to reach the limit, while in age group 3, this percentage increased to about 78%. Differences due to gender were not significant when comparing the effective recognition values.

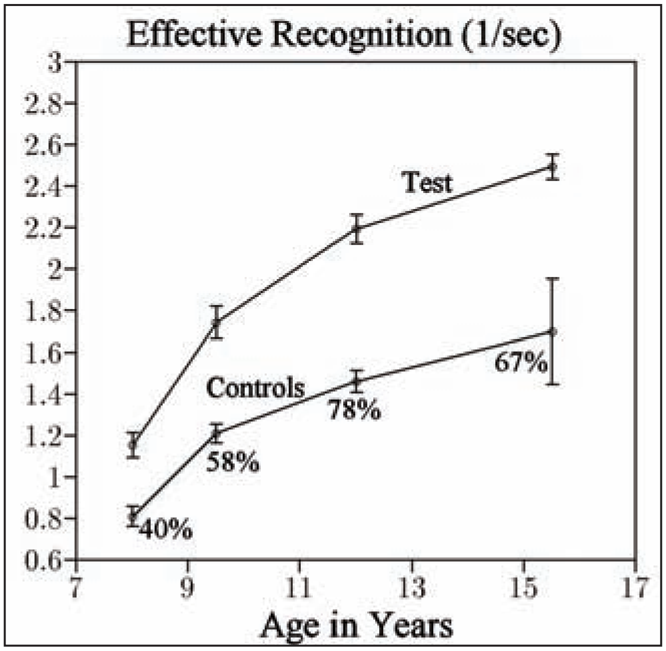

Figure 6: *The effective recognition as a function of age for both groups of subjects. The numbers indicate the percentage of test subjects performing below the 16-percentile of the age matched control groups.*

The further analysis shows that the use of small item numbers also differentiated between the groups. The percentage of correct responses divided by the corresponding reaction time averaged for item numbers 2, 3 and 4 was calculated. The developmental deficits obtained by the comparison of the age curves showed also increase with age in a very similar way as indicated by the variable shown in Fig.6. In other words: even when dealing with small item numbers test children performed below the level of the controls at any age tested in this study.

Discussion

The present study shows that the capacity of counting one up to eight briefly presented items is lower for children with problems in acquiring basic arithmetic skills as compared with age matched controls. The deficits occur in all age groups and increase with age. The responses of the test subjects are slower and more often incorrect even for item numbers below 3 or 4. Given that most test children were classified as dyscalculic by their schools, one may conclude that in dyscalculia the specific visual capacity of subitizing and counting exhibits developmental deficits in 40 to 78% of the cases depending on age.

From PET studies it is known that the extrastriate middle occipital and the parietal cortical areas are involved in subitizing and counting[4]. Since the stimuli in the present study were presented only briefly, it was impossible to count the items by scanning them by saccades. The items were seen by parafoveal vision and were stored in visual memory. The subitizing and/or counting process rely on the effectiveness of storing and reading the data. A deficit in this function may contribute to the problems of dyscalculia subjects, because they could not use this visual memory function as effectively as other children. These children were supposed to develop a sense of number,: the inner representation of "how many items" there are and which is meant by the number word "five" and/or by the digit "5" was not as immediately available for them as for normal children.

At this point the idea of a causal relationship between dyscalculia and subitizing must still be considered as an hypothesis. However, similar to other sensory and optomotor functions subitizing and counting may be improved by specific training procedures. Whether a successful training results in an increase in acquiring basic arithmetic skills will be considered in the following paper.

It might be interesting to compare the present results with those of dyslexic subjects, who were diagnosed for deficits in saccade control and low level auditory discrimination. First of all the temporal extend of the development is very similar lasting also until adulthood[8, 17]. Second, dyslexics exhibit deficits in one or both of the optomotor[18] and auditory domains[9]. Specific daily practice of saccade control and/or low level auditory functions reduces the developmental deficits in up to 70 to 85% of cases[19, 20] and leads to increases in the process of acquiring reading and spelling skills, respectively[20]. Preliminary data from our laboratory show, that up to 60% of dyslexic children also suffer from deficits in subitizing and counting and dyscalculia children may also suffer from deficits in saccade control.

References

1. Cowan N. The magical number 4 in short term memory: A reconsideration of mental storage capacity. Behav Brain Sci 2001;26:87 116.

2. Trick LM and Phylyshyn ZW. Why are small and large numbers enumerated differently? A limited capacity preattentive stage in vision. Psychol Rev 1994;101:80 102.

3. Mandler G and Shebo BJ. Subitizing: An analysis of its component processes. J Exp Psychol Gen 1982;11:1 22.

4. Piazza M, Mechelli A, Butterworth B, and Price CJ. Are subitizing and counting implemented as separate or functionally overlapping pocesses? Neuroimage 2002;15:435 46.

5. Balakrishnan JD and Ashby FG. Is subitizing a unique numerical ability? Percept Psychophys 1991;50:555 64.

6. Simon T and Vaishsnavi S. Subitizing and counting depend on different attentional mechnisms: Evidence from visual enumeration in afterimages. Percept Psychophys 1996;58:915 26.

7. Dehaene S, Piazza M, Pinel P, and Cohen L. Three parietal circuits for number processing. Cogn Neuropsychology 2003;20:487 506.

8. Fischer B, Biscaldi M, Gezeck S. On the development of voluntary and reflexive components in human saccade generation. Brain Res 1997;754:285 97.

9. Fischer B and Hartnegg K. On the development of low level auditory discrimination and deficits in dyslexia. Dyslexia 2004;10(2):105 118

10. Dehaene S. Der Zahlensinn. Basel: Birkhäuser Verlag; 1999.

11. Koontz KL and Berch DB. Identifying simple numerical stimuli: processing inefficiencies exhibited by arithmetic learning disabled children. Mathematical Cognition 1996;2:1 24.

12. Landerl K, Bevan A, and Butterworth B. Developmental dyscalculia and basic numerical capacities: a study of 8 9 year old students. Cognition 2004;93:99 125.

13. Kucian K, Loenneker T, Dietrich T, Dosch M, Martin E, and von Aster M. Impaired neural networks for approximate caculation in dyscalculic children: a functional MRI study. Behavv Brain Funct 2006:2:31 48.

14. Fischer B, Ramsperger E. Human express saccades: extremely short reaction times of goal directed eye movements. Exp Brain Res 1984;57:191 5.

15. Jensen EM, Reese EP, and Reese TW. The subitizing and counting of visually presented fields of dots. J Psychology: Interdisciplinary & Applied 1950:30;363 92.

16. Sathian K, Simon TJ, Peterson S, Patel GA, Hoffman JM, and Grafton ST. Neural evidence linking visual object enumeration and attention. J Cogn Neurosci 1999;11:36 51.

17. Fischer B, Hartnegg K. On the development of low level auditory discrimination. www.dyslexia-adults.com/auditory.html. 2001

18. Biscaldi M, Fischer B, Hartnegg K. Voluntary saccade control in dyslexia. Perception 2000;29:509 21.

19. Fischer B and Hartnegg K. Effects of visual training on saccade control in dyslexia. Perception 2000;29(5):531 42.

20. Schäffler T, Sonntag J, and Fischer B. The effect of daily practice on low level auditory discrimination, phonological skills, and spelling in dyslexia. Dyslexia 2004;10(2):119 30.